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WAVE GENERATED OSCILLATORY CURRENTS ALONG THE BOTTOM IN THE EUL--ETC(U)
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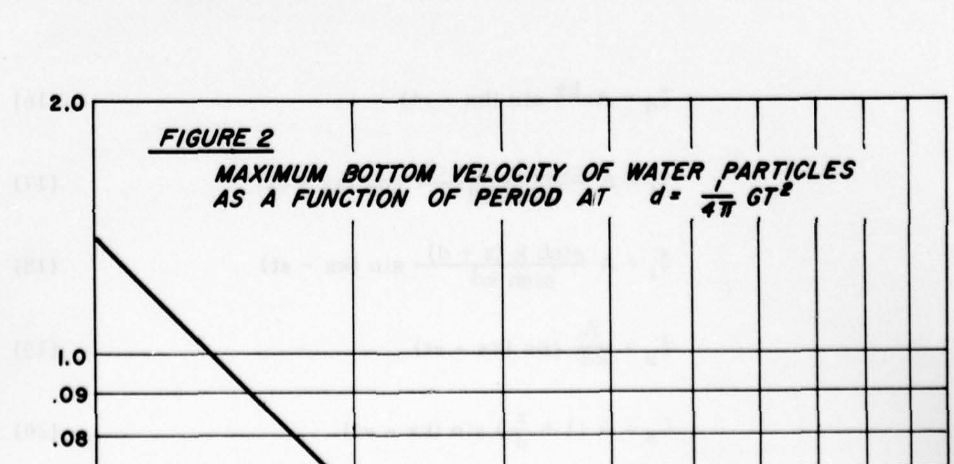

FIGURE 2

MAXIMUM BOTTOM VELOCITY OF WATER PARTICLES AS A FUNCTION OF PERIOD AT $d = \frac{1}{4\pi} GT^2$

LEVEL II

1

6

Wave Generated Oscillatory Currents Along the Bottom in the Eulittoral and Sublittoral Zones

(With graphs for determining maximum horizontal velocity, maximum displacement, and mean acceleration.)

12 31p.

10 Lee M. Hunt

11 May 1961

Mine Advisory Committee

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JOB

CONTENTS

LIST OF SYMBOLS

A	=	wave amplitude = $\frac{H}{2}$
a	=	mean particle acceleration
C	=	shallow water phase or wave velocity
C_o	=	deep water phase or wave velocity
D	=	diameter of particle orbit
d	=	depth of water, measured from still water level to the bottom; taken as positive downward
e	=	base of Naperian logarithms = 2.718
g	=	acceleration of gravity - 32.2 ft/sec^2
H	=	shallow water wave height
H_o	=	deep water wave height
k	=	wave number = $\frac{2\pi}{L}$
L	=	shallow water wave length
L_o	=	deep water wave length
n	=	ratio of group velocity to phase velocity
T	=	wave period
t	=	time
U_d	=	deep water horizontal particle velocity
U_i	=	intermediate water horizontal particle velocity
U_s	=	shallow water horizontal particle velocity

U_{max} = maximum horizontal particle velocity

W_d = deep water vertical particle velocity

W_i = intermediate water vertical particle velocity

W_s = shallow water vertical particle velocity

x = horizontal coordinate (arbitrary origin), positive in direction of wave advance

z = depth below still-water level; taken as negative downward

ξ_d = deep water horizontal displacement of particle from its mean position

ξ_i = intermediate water horizontal displacement of particle from its mean position

ξ_s = shallow water horizontal displacement of particle from its mean position

ξ_{max} = maximum horizontal displacement of particle from its mean position

ζ_d = deep water vertical displacement of particle from its mean position

ζ_i = intermediate water vertical displacement of particle from its mean position

ζ_s = shallow water vertical displacement of particle from its mean position

ζ_{max} = maximum vertical displacement of particle from its mean position

π = 3.1416

σ = angular frequency = $\frac{2\pi}{T}$

LIST OF FIGURES AND TABLES

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ABSTRACT

INTRODUCTION

Wave generated oscillatory currents along the ocean floor provide one of the more important forces participating in the erosion, transportation, and deposition of marine sediments. These currents are especially active in the eulittoral zone which ranges from the high tide level to a depth of 150 feet. Significant currents in the sublittoral zone, which continues from the lower border of the eulittoral zone to a depth of 600 feet or the edge of the continental shelf, are generated only by exceptionally long period waves. The effect of these currents has long been of importance to marine geologists and coastal engineers, and their importance can only be enhanced by the increasing amount of instruments and installations being placed on the ocean bottom in these zones. ABSTRACT

Due to turbulence as well as the oscillatory nature of wave generated bottom currents, attempts to measure their magnitudes have met with considerable difficulty. Since observations made in controlled laboratory tests and in some field tests show reasonable agreement between actual and theoretical particle motion[1,2,3,4] these currents are usually calculated from theory. This report discusses briefly the theory upon which these calculations are based and then, in order to facilitate the use of the theory, provides graphs of maximum particle velocity, maximum particle displacement, and mean particle acceleration at the bottom as a function of depth, wave period, and wave height. The curves are drawn such that if the latter three variables are known the former three can be read directly from the graphs. If maximum particle velocity and displacement over the entire water column is required the reader is referred to GRAPHS FOR OBTAINING ORBITAL DISPLACEMENTS AND VELOCITIES, Scripps Institution of Oceanography, Wave Project No. 71, Nov. 12, 1947.

In using the graphs it should be kept in mind that the theory does not take into consideration the spectral nature of some naturally occurring waves in shallow water. The values presented then, represent a somewhat simplified picture of current characteristics under these conditions. It might be pointed out, however, that due to the filtering out of waves with periods smaller than the one considered, bottom currents calculated from theory are more accurate than those higher in the water column.

CHARACTERISTICS OF ORDINARY GRAVITY WAVES TRANSITING FROM "DEEP" TO "SHALLOW" WATER

Ordinary Gravity Waves

Waves having periods ranging from 1 to 30 seconds are known as ordinary gravity waves as distinguished from ultra-gravity waves with a period range of 0.1 to 1 second, and infra-gravity waves whose period range is 30 seconds to 5 minutes. Ordinary gravity waves include the wind generated waves known as seas while under the direct influence of the wind, and swell after they have begun to decay in a region of lighter wind or calm. Seas generally have a shorter period than swell with the division being at around 9 seconds. There is, however, some overlap. This whole band of ordinary gravity waves contains a large fraction of the total wave energy and is, therefore, important to a variety of problems.

Ordinary gravity waves are generated by the pressure and tangential stresses applied to the water surface by wind action. Dimensional characteristics, therefore, depend upon wind velocity, duration, and fetch as well as the distance the waves have traveled from their area of generation. This last point is due to the decay undergone by waves once the generating force is removed. Decay may be a slow process as shown by the summer swell which breaks on the California coast after traveling more than 4000 miles from their generation area in the "Roaring Forties" and "Furious Fifties" of the South Pacific.

Transition from "Deep" to "Shallow" Water Waves

Ordinary gravity waves may be divided into deep (short) and shallow (long) water waves through a consideration of the relationship between phase velocity, wave length, and water depth. The phase velocity can be evaluated with sufficient accuracy from the classical equation for gravity waves of small steepness[1,2,5]:

$$C^2 = \frac{gL}{2\pi} \tanh kd. \qquad (1)$$

If the relative depth (d/L) is greater than 0.5, then the hyperbolic tangent can be replaced by unity to an accuracy of one per cent. Equation (1) is thereby reduced to

$$C^2 = \frac{gL}{2\pi}. \tag{2}$$

Waves under conditions where d/L is greater than 0.5 and where the phase velocity is independent of depth are called deep water waves, the wave length of which is given by

$$L_o = \frac{g}{2\pi} T^2. \tag{3}$$

If, on the other hand, d/L is less than 0.05, the hyperbolic tangent can be replaced by the argument $2\pi d/L$ to the same accuracy and equation (1) becomes

$$C^2 = gd. \tag{4}$$

Waves under these conditions are independent of wave length, but dependent upon depth, and are called shallow water waves.

Waves whose relative depth lies between 0.05 and 0.5 are termed intermediate and equation (1) must be used.

Throughout the remainder of this paper, unless otherwise specified, shallow water will be used to indicate any relative depth less than the depth at the deep to shallow water wave transition point as given by

$$d = \frac{1}{4\pi} gT^2 \tag{5}$$

or $1/2\ L_o$.

Orbital Motion of Water Particles

Associated with the motion of waves is a motion of the water particles themselves. In the deep water case this motion is in the form of circles in

the vertical plane with the water particles moving in the direction of wave propagation under the wave crest and in the opposite direction under the trough. The time required for one complete orbit is equal to the period of the wave. The orbital diameter is given by

$$D = He^{kz}, \tag{6}$$

hence D decreases exponentially with increasing depth. The following rule is often useful: The orbital diameter, (equal to the wave height at the surface), is reduced by one-half with each depth increase equal to one-ninth of the wave length.

As an example a deep water wave having dimensions $T = 10$ seconds, $L_o = 512$ feet, and $H = 22$ feet will have a surface orbit of 22 feet in diameter. At a depth equal to one-half the wave length (256 feet) the orbital diameter is 0.95 feet, and at a depth equal to the wave length (512 feet) the orbital diameter is 0.042 feet. Since the orbital diameter at $1/2\ L_o$ is only 1/23 that of the surface orbit it can be seen that the bottom can have no significant effect on the character of the waves as long as d exceeds $1/2\ L_o$.

As any wave crosses its particular transition point from deep to shallow water, an increasing *potential orbital* diameter is brought in contact with the bottom. However, at the sea bed, the component of motion normal to the bottom must vanish, and the response of the water particles over the entire water column to this transition is a tendency toward elliptical rather than circular orbits. Moreover, the eccentricity of the elliptical orbits increases with decreasing depth.

Particle Velocity, Displacement, and Acceleration

The speed with which water particles move around their orbits — essentially uniform while they are circular — is no longer so after they are transformed into ellipses, but is greatest near the crest and trough of the wave. This discrepancy between the speeds along different parts of the elliptical orbit increases with decreasing depth, since it is proportional to the length of the major axis of the ellipse. Since the transformation of the orbit from circle to ellipse consists of an expansion of the horizontal axis, with the vertical axis changing only as much as the height of the waves, the speed with which the water particle advances in the crest and recedes in the trough grows greater as the depth decreases.

Figures 1 and 2 graphically illustrate the relative depth of transition as a function of period, the depth range over which the maximum bottom particle velocity at the transition point is exceeded, and the relationship between maximum particle velocity at the transition point and wave period.

Horizontal and vertical particle velocity for deep, intermediate, and shallow water are given by

$$U_d = A\sigma^{kz} \cos (kx - \sigma t) \tag{7}$$

$$W_d = A\sigma^{kz} \sin (kx - \sigma t) \tag{8}$$

$$U_i = A\sigma \frac{\cosh k (z + d)}{\sinh kd} \cos (kx - \sigma t) \tag{9}$$

$$W_i = A\sigma \frac{\sinh k (z + d)}{\sinh kd} \sin (kx - \sigma t) \tag{10}$$

$$U_s = \frac{A\sigma}{kd} \cos (kx - \sigma t) \tag{11}$$

$$W_s = A\sigma (1 + \frac{z}{d} \sin (kx - \sigma t) \tag{12}$$

respectively. The above equations apply for a water level profile which varies as $A \cos (kx - \sigma t)$. At the bottom, of course, $-z$ is equal to d, and W becomes O. If only the maximum velocity at the bottom is considered ($x, t = o$) equations (9) and (11) may be rewritten as

$$U_{imax} = A\sigma \frac{1}{\sinh kd} \tag{13}$$

$$U_{smax} = \frac{A\sigma}{kd} = A\sqrt{\frac{g}{d}}. \tag{14}$$

Particle displacement is taken as the displacement in one direction of a particle from its mean position. Twice the maximum displacement then is the total range of particle excursion. The horizontal and vertical displacement is given by

$$\xi_d = A_e^{kz} \cos (kx - \sigma t) \tag{15}$$

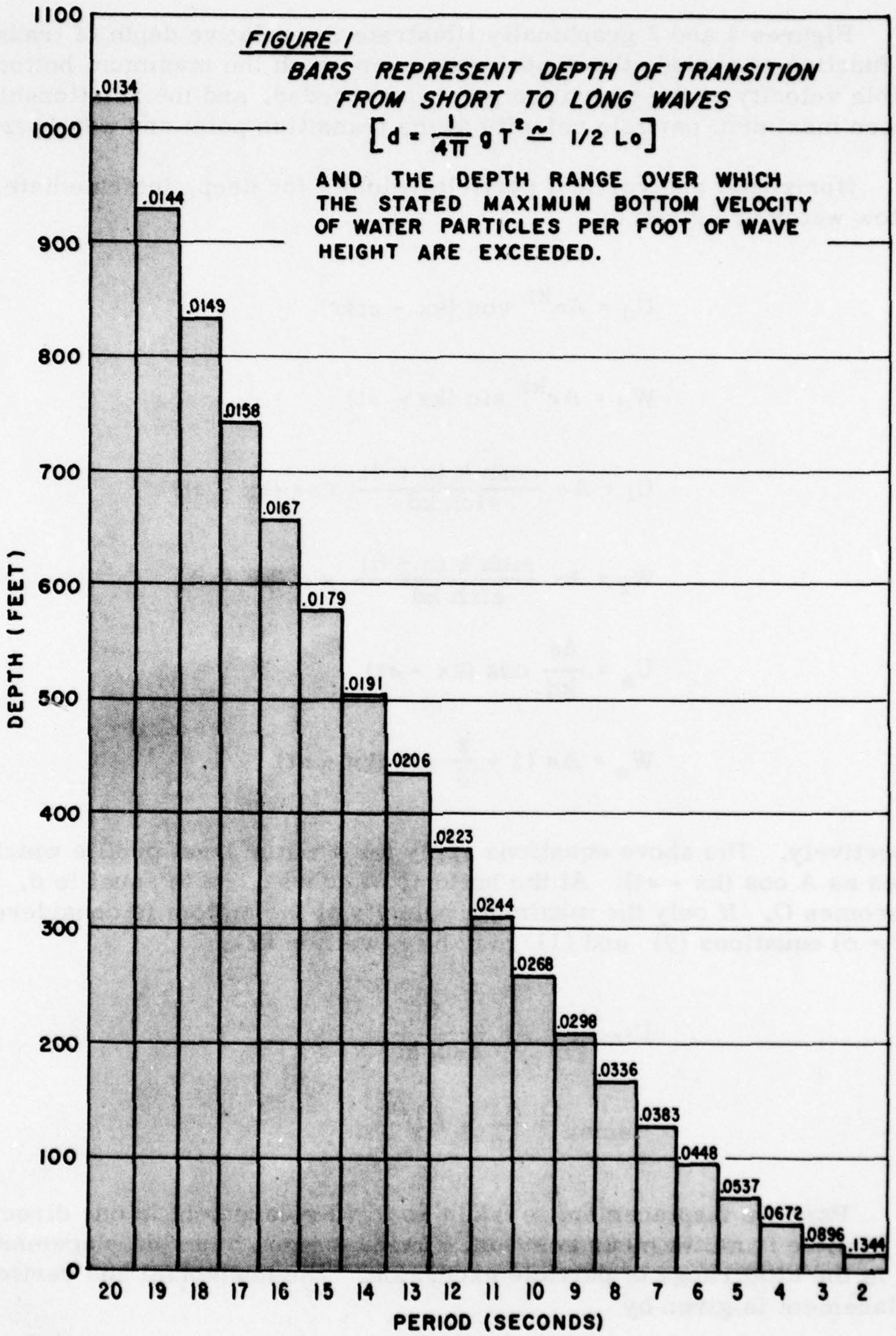
FIGURE 1
BARS REPRESENT DEPTH OF TRANSITION FROM SHORT TO LONG WAVES
[d = 1/4π gT² ≃ 1/2 Lo]
AND THE DEPTH RANGE OVER WHICH THE STATED MAXIMUM BOTTOM VELOCITY OF WATER PARTICLES PER FOOT OF WAVE HEIGHT ARE EXCEEDED.
DEPTH (FEET)
PERIOD (SECONDS)
1100
1000
900
800
700
600
500
400
300
200
100
0
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
.0134
.0144
.0149
.0158
.0167
.0179
.0191
.0206
.0223
.0244
.0268
.0298
.0336
.0383
.0448
.0537
.0672
.0896
.1344

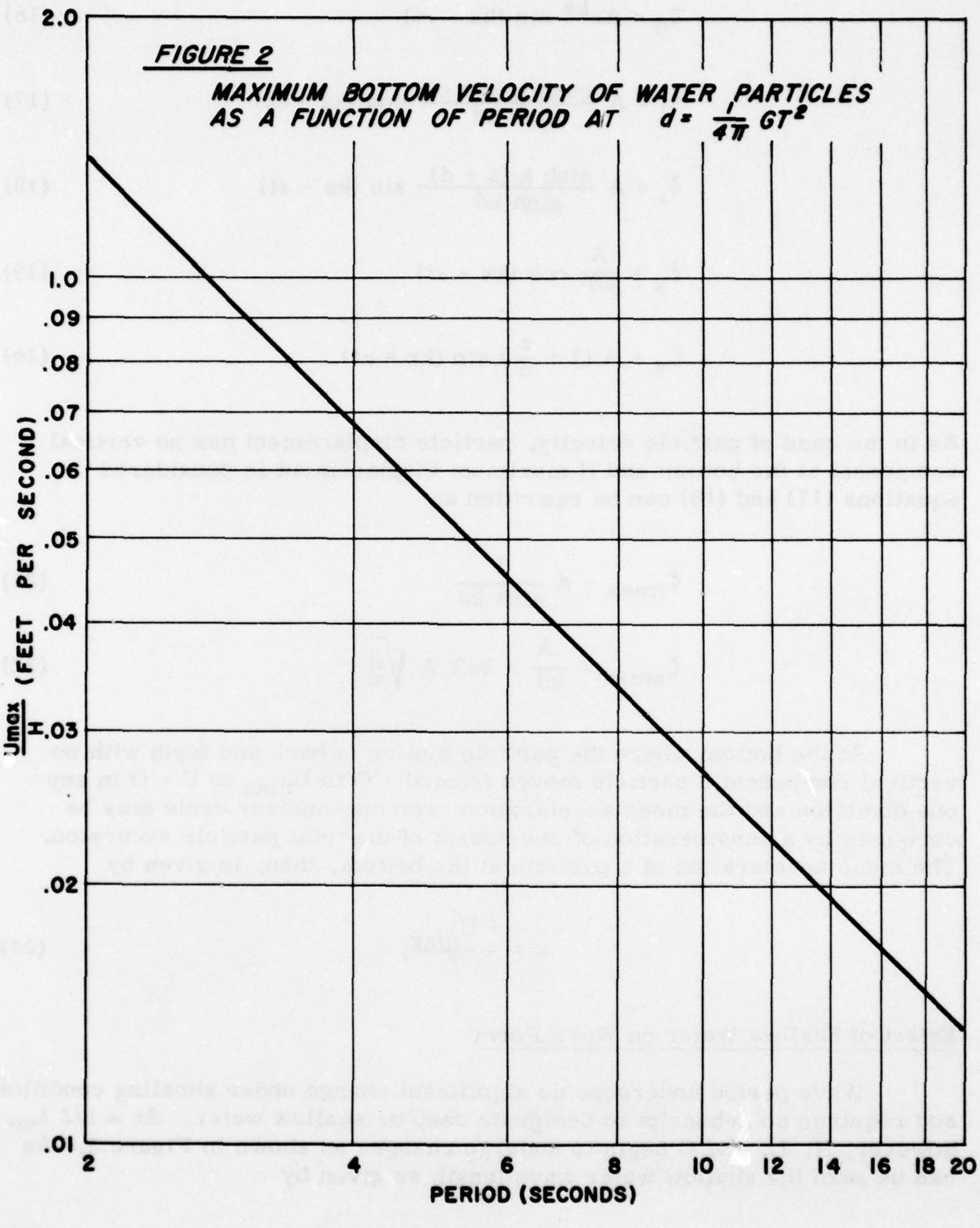
FIGURE 2
MAXIMUM BOTTOM VELOCITY OF WATER PARTICLES
AS A FUNCTION OF PERIOD AT $d = \frac{1}{4\pi} GT^2$
2.0
1.0
.09
.08
.07
.06
.05
.04
.03
.02
.01
$\frac{U_{max}}{H}$ (FEET PER SECOND)
2
4
6
8
10
12
14
16
18
20
PERIOD (SECONDS)

$$\zeta_d = Ae^{kz} \sin (kx - \sigma t) \tag{16}$$

$$\xi_i = A \frac{\cosh k(z+d)}{\sinh kd} \cos (kx - \sigma t) \tag{17}$$

$$\zeta_i = A \frac{\sinh k(z+d)}{\sinh kd} \sin (kx - \sigma t) \tag{18}$$

$$\xi_s = \frac{A}{kd} \cos (kx - \sigma t) \tag{19}$$

$$\zeta_s = A \left(1 + \frac{z}{d}\right) \sin (kx - \sigma t). \tag{20}$$

As in the case of particle velocity, particle displacement has no vertical component at the bottom and if maximum displacement is considered equations (17) and (19) can be rewritten as

$$\xi_{imax} = A \frac{1}{\sinh kd} \tag{21}$$

$$\zeta_{smax} = \frac{A}{kd} = 2\pi T A \sqrt{\frac{g}{d}}. \tag{22}$$

At the bottom where the particle motion is back and forth with no vertical component a particle moves from $U = O$ to U_{max} to $U = O$ in any one direction and the mean acceleration over one-quarter cycle may be computed by a consideration of one-fourth of the total particle excursion. The mean acceleration of a particle at the bottom, then, is given by

$$a = \frac{4 U_{max}}{T}. \tag{23}$$

Effect of Shallow Water on Wave Form

Wave period undergoes no significant change under shoaling conditions and requires no subscript to designate deep or shallow water. At $\approx 1/2\ L_o$, however, H, L, and C begin to undergo changes as shown in Figure 3. As can be seen the shallow water wave length as given by

$$L = L_o \tanh kd \tag{24}$$

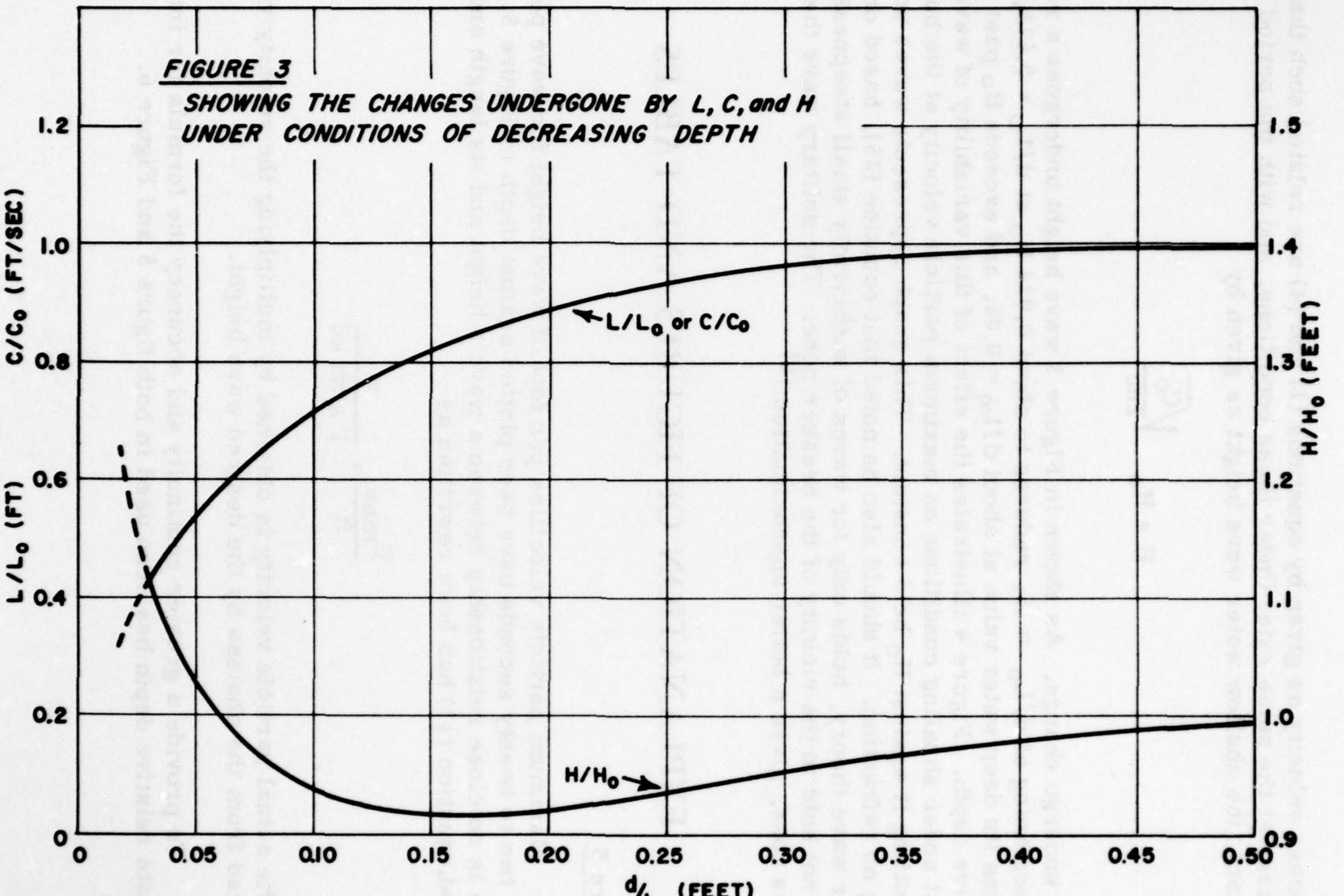

FIGURE 3

SHOWING THE CHANGES UNDERGONE BY L, C, and H UNDER CONDITIONS OF DECREASING DEPTH

and phase velocity as given by equations (1) and (4) are related such that they decrease at the same rate. Under these conditions, and with the period being constant, the shallow water wave height as given by

$$H = H_o \sqrt{\frac{C_o}{2nc}} \tag{25}$$

must undergo change. As shown in Figure 3 wave height undergoes a reduction beginning at $d/L_o \simeq .5$, reduces to about 0.914 H_o at $d/L_o = 0.615$, regains its deep water value at about $d/L_o = 0.06$, and exceeds H_o past this relative depth. Figure 4 illustrates the effect of this variability of wave height under shoaling conditions on maximum particle velocity at the bottom by plotting it against H_o held constant. This graph represents waves undergoing no refraction. It should also be noted that equation (25), based on linear wave theory, holds only for waves of sufficiently small steepness and does not hold in the vicinity of the breaker zone. The solitary wave theory, in this zone, gives a better approximation.[6]

EXPLANATION OF FIGURES AND TABLES

Figure 5

Maximum particle velocities per foot of wave height for wave periods from two to twenty seconds have been plotted against depth in Figure 5. Since there is no close relationship between a wave's height and its length and period,equation (13) has been rewritten as

$$\frac{U_{max}}{H} = \frac{\pi}{T \sinh kd} \tag{26}$$

and the actual particle velocity is obtained by multiplying the velocity value as read from the absissa by the desired wave height.

To provide a greater continuity and accuracy the formula for intermediate relative depth has been used in both figure 5 and Figure 6.

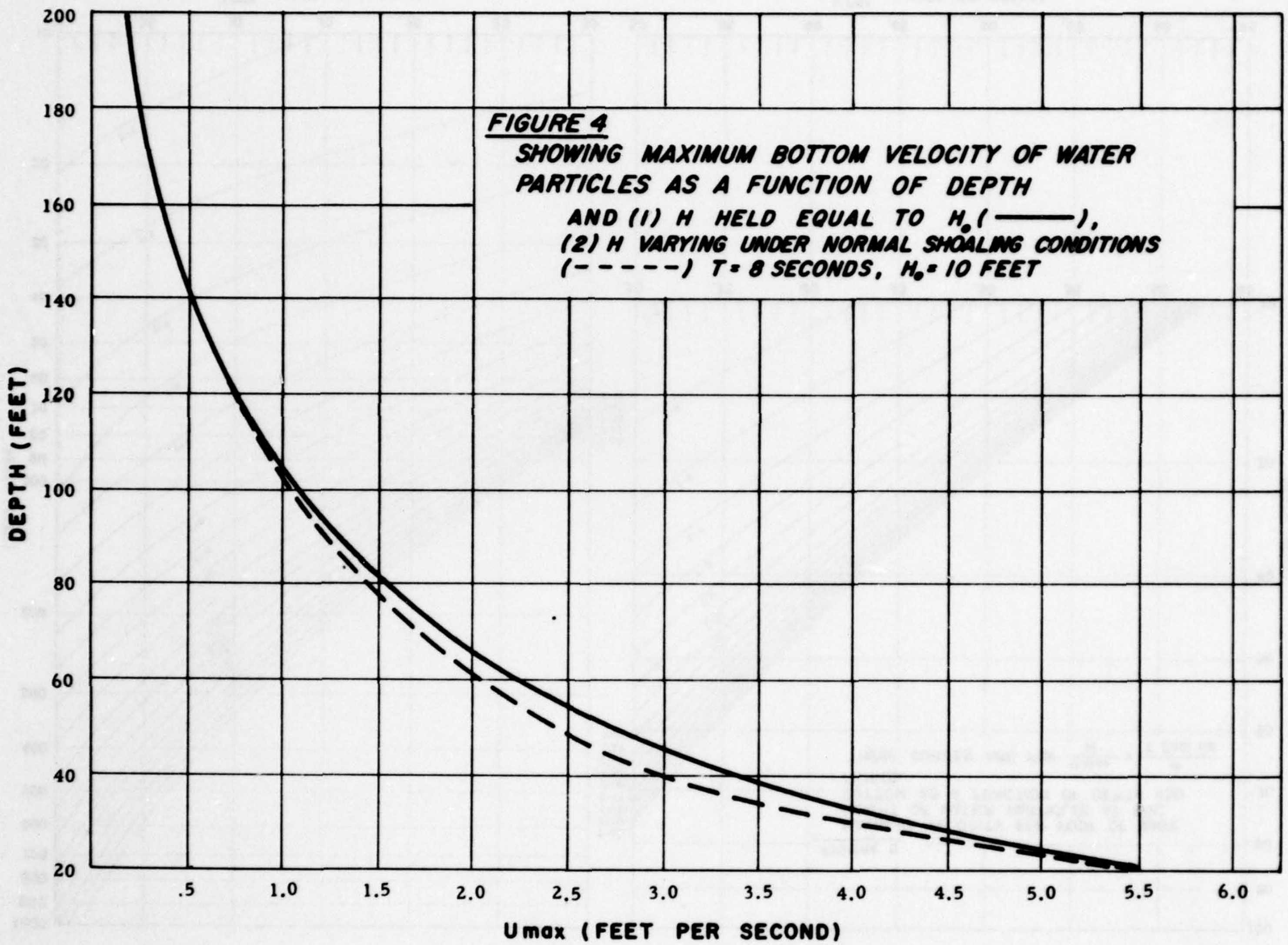

FIGURE 4

SHOWING MAXIMUM BOTTOM VELOCITY OF WATER PARTICLES AS A FUNCTION OF DEPTH AND (1) H HELD EQUAL TO H_o (———), (2) H VARYING UNDER NORMAL SHOALING CONDITIONS (- - - - - -) T = 8 SECONDS, H_o = 10 FEET

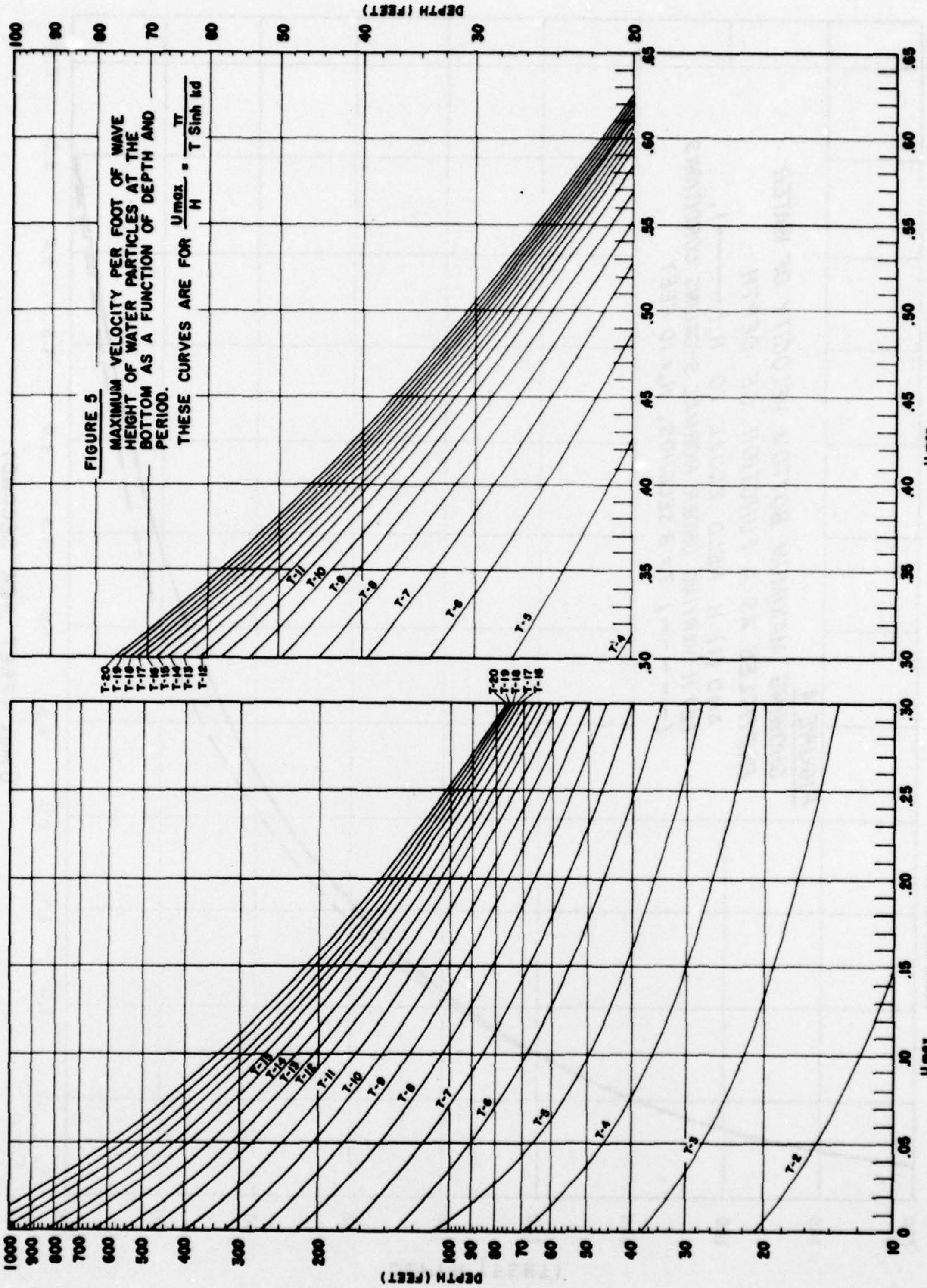
FIGURE 5
MAXIMUM VELOCITY PER FOOT OF WAVE HEIGHT OF WATER PARTICLES AT THE BOTTOM AS A FUNCTION OF DEPTH AND PERIOD.
THESE CURVES ARE FOR $\frac{U_{max}}{H} = \frac{\pi}{T \sinh kd}$
DEPTH (FEET)
$\frac{U_{max}}{H}$ (FEET PER SECOND)
T-2
T-3
T-4
T-5
T-6
T-7
T-8
T-9
T-10
T-11
T-12
T-13
T-14
T-15
T-16
T-17
T-18
T-19
T-20

Figure 6

Maximum particle displacement per foot of wave height for periods from two to twenty seconds have been plotted against depth in Figure 6. As in the case of particle velocity equation (21) has been rewritten as

$$\frac{\xi_{max}}{H} = \frac{1}{2 \sinh kd} \tag{27}$$

and maximum particle displacement is obtained by multiplying the absissa value by the desired wave height.

Figure 7

In considering the effect of bottom particle velocity on sediments it is often desirable to know the velocity distribution over the path of particle excursion. The time range, for instance, over which a particle has exceeded a particular velocity is often useful. In order to facilitate the calculation of a velocity curve for a desired height, period, and depth the mean acceleration per foot of wave height for periods from two to twenty seconds have been plotted against depth in Figure 7. As in Figures 5 and 6 the absissa value is multiplied by the desired wave height.

Table I

The particle velocity, displacement, and acceleration values used in plotting the various curves for each depth and period used are listed in Table I.

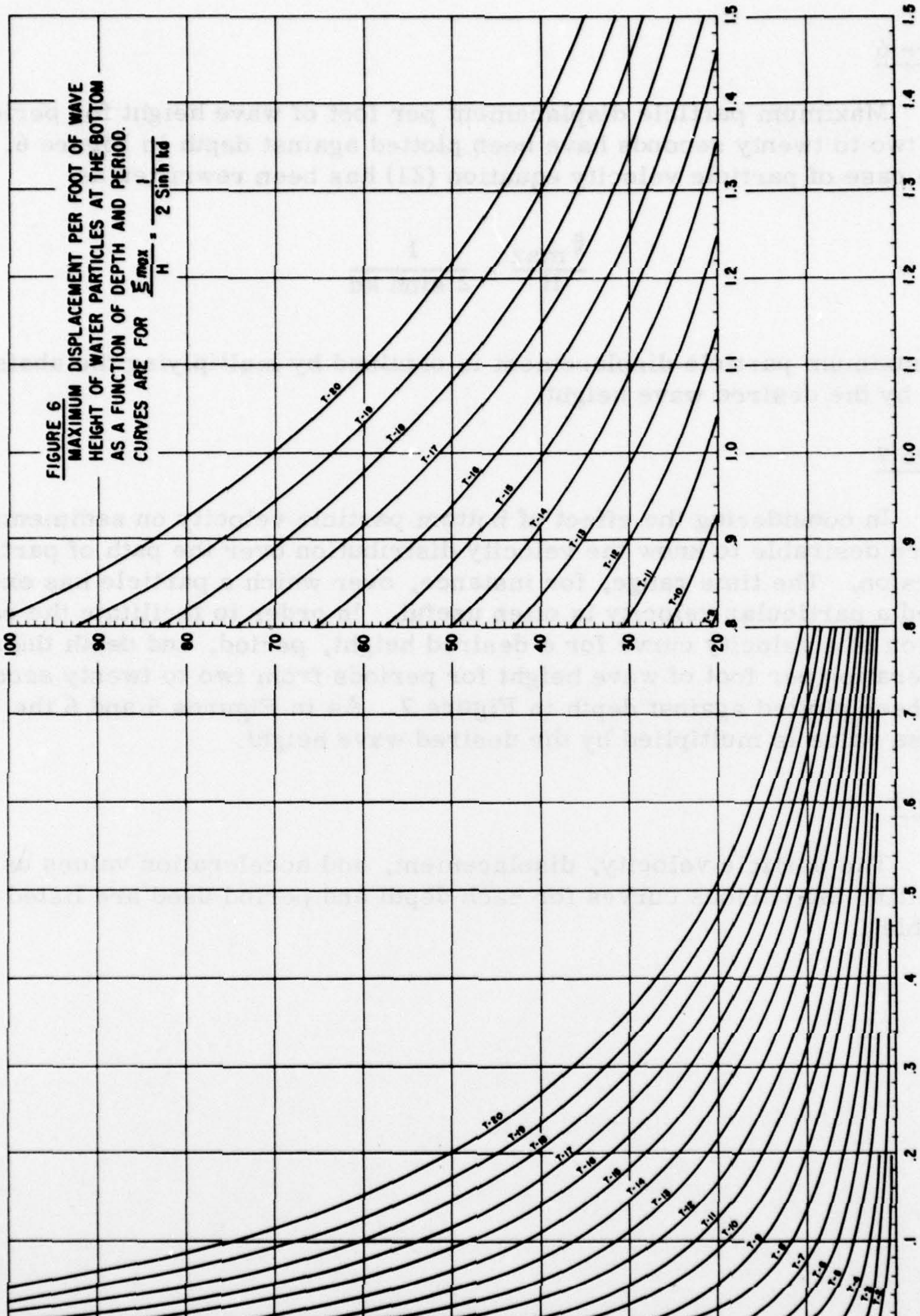

FIGURE 6
MAXIMUM DISPLACEMENT PER FOOT OF WAVE HEIGHT OF WATER PARTICLES AT THE BOTTOM AS A FUNCTION OF DEPTH AND PERIOD.
CURVES ARE FOR $\frac{\xi_{max}}{H} = \frac{1}{2 \sinh kd}$

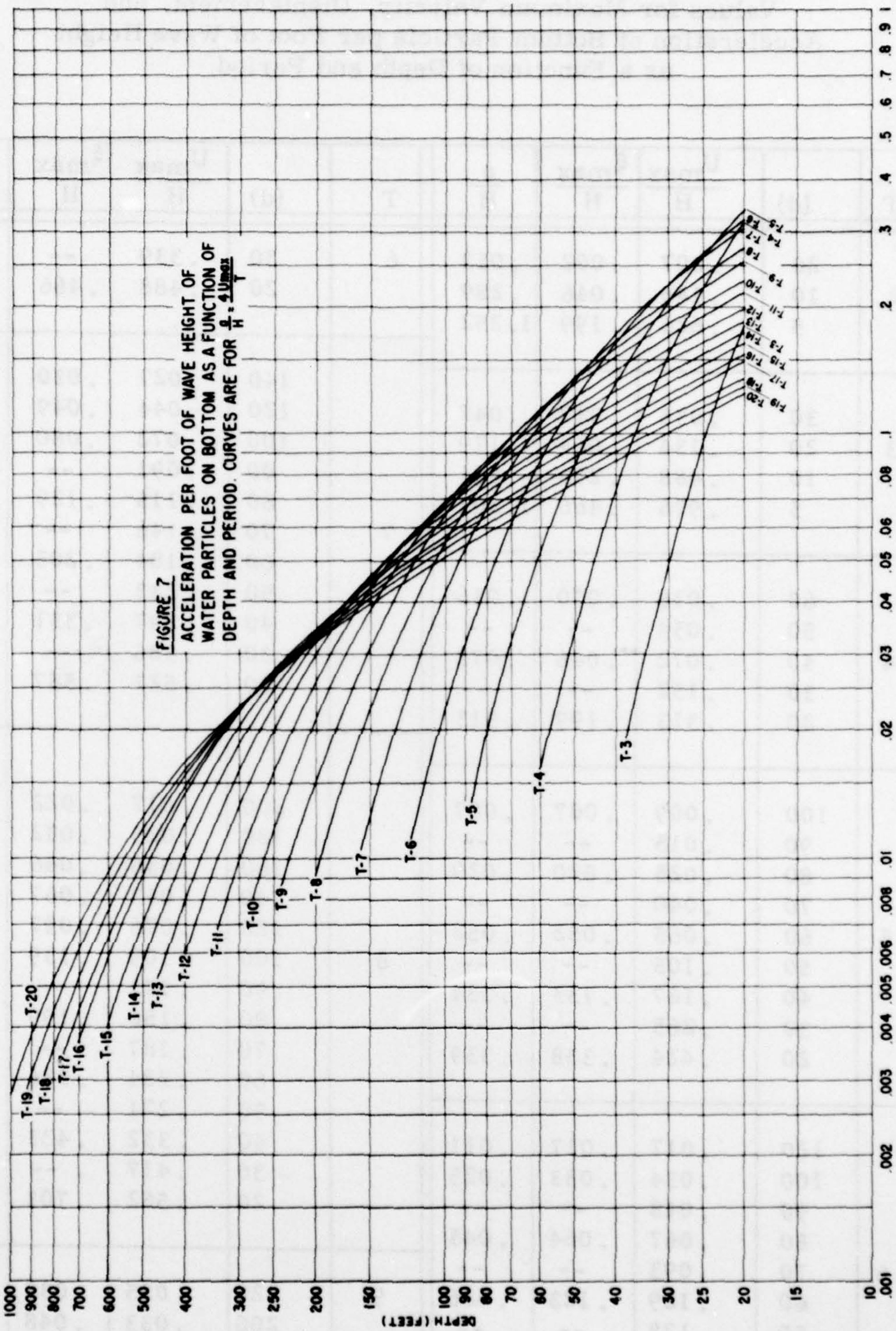

FIGURE 7
ACCELERATION PER FOOT OF WAVE HEIGHT OF WATER PARTICLES ON BOTTOM AS A FUNCTION OF DEPTH AND PERIOD. CURVES ARE FOR $\frac{a}{H} = \frac{4 U_{max}}{T}$

TABLE I

Values for Maximum Velocity, Displacement, and Acceleration of Bottom Particle per Foot of Wave Height as a Function of Depth and Period

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
	20	.007	.002	.013
2	10	.145	.046	.289
	5	.626	.199	1.252
	30	.035	.017	.047
3	20	.134	.064	.179
	10	.488	.233	.651
	5	.976	.466	1.301
	60	.016	.010	.016
	50	.034	--	--
4	40	.072	.046	.072
	30	.152	--	--
	20	.313	.199	.313
	100	.009	.007	.007
	90	.015	--	--
	80	.025	.020	.020
	70	.040	--	--
5	60	.065	.052	.052
	50	.105	--	--
	40	.167	.133	.134
	30	.265	--	--
	20	.424	.338	.339
	120	.017	.017	.011
	100	.034	.033	.023
	90	.048	--	--
	80	.067	.064	.045
6	70	.093	--	--
	60	.129	.123	.086
	50	.178	--	--
	40	.244	.233	.163
6	30	.339	--	--
	20	.488	.466	.325
	140	.027	.030	.015
	120	.044	.049	.025
	100	.072	.080	.041
	90	.091	--	--
	80	.115	.129	.066
7	70	.146	--	--
	60	.184	.205	.105
	50	.233	--	--
	40	.297	.331	.170
	30	.386	--	--
	20	.527	.587	.301
	200	.017	.022	.009
	180	.025	.032	.013
	160	.036	.046	.018
	140	.053	.067	.027
	120	.076	.097	.038
8	100	.109	.139	.055
	90	.131	--	--
	80	.156	.199	.078
	70	.187	--	--
	60	.224	.286	.112
	50	.271	--	--
	40	.332	.423	.166
	30	.417	. --	--
	20	.552	.703	.276
9	220	.025	.035	.011
	200	.033	.048	.015
	180	.045	.064	.020

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
9	160	.060	.086	.027
	140	.080	.115	.036
	120	.107	.153	.048
	100	.141	.203	.063
	90	.163	--	--
	80	.188	.270	.084
	70	.218	--	--
	60	.254	.364	.113
	50	.298	--	--
	40	.356	.511	.158
	30	.438	--	--
	20	.569	.816	.253
10	280	.020	.032	.008
	260	.026	.041	.010
	240	.033	.052	.013
	220	.041	.066	.016
	200	.053	.084	.021
	180	.066	.106	.026
	160	.084	.133	.034
	140	.105	.168	.042
	120	.132	.211	.053
	100	.167	.266	.067
	90	.188	--	--
	80	.212	.338	.085
	70	.240	--	--
	60	.275	.438	.110
	50	.318	--	--
	40	.374	.596	150
	30	.454	--	--
	20	.582	.927	.233
11	320	.022	.039	.008
	300	.027	.047	.010
	280	.033	.058	.012
	260	.040	.070	.015
	240	.049	.085	.018
	220	.059	.104	.021
	200	.072	.125	.026
	180	.087	.152	.032
	160	.105	.183	.038
	140	.127	.222	.046
	120	.153	.268	.056
	100	.186	.327	.068
	90	.207	--	--
	80	.230	.403	.084
	70	.258	--	--
	60	.291	.509	.106
	50	.332	--	--
	40	.387	678	.141
	30	.465		--
	20	.592	1.036	.215
12	400	.017	.033	.006
	380	.020	.039	.007
	360	.024	.046	.008
	340	.028	.054	.009
	320	.034	.064	.011
	300	.040	.076	.013
	280	.047	.089	.016
	260	.055	.105	.018
	240	.065	.123	.022
	220	.076	.145	.025
	200	.089	.170	.030
	180	.104	.200	.035
	160	.122	.233	.041
	140	.144	.275	.048
	120	.170	.324	.057
	100	.202	.386	.067
	90	.221	--	--
	80	.244	.466	.081
	70	.271	--	--
	60	.303	.579	.101
	50	.343	--	--
	40	.397	.759	.132

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
12	30	.474	--	--
	20	.598	1.143	.199
	460	.017	.035	.005
	440	.020	.039	.006
	420	.023	.047	.007
	400	.026	.054	.008
	380	.030	.062	.009
	360	.035	.072	.011
	340	.040	.082	.012
	320	.046	.095	.014
	300	.053	.109	.016
	280	.060	.125	.018
	260	.069	.143	.021
	240	.079	.164	.024
	220	.091	.188	.028
13	200	.104	.215	.032
	180	.119	.246	.037
	160	.137	.283	.042
	140	.158	.326	.049
	120	.183	.378	.056
	100	.214	.443	.066
	90	.233	--	--
	80	.255	.527	.078
	70	.281	--	--
	60	.312	.647	.096
	50	.352	--	--
	40	.405	.838	.125
	30	.480	--	--
	20	.603	1.249	.186
14	520	.017	.038	.005
	500	.019	.043	.005
	480	.022	.049	.006
	460	.025	.055	.007
	440	.028	.063	.008
	420	.032	.071	.009
	400	.036	.080	.010
	380	.040	.090	.011
	360	.045	.101	.013
	340	.051	.114	.015
	320	.058	.129	.017
	300	.065	.145	.019
	280	.073	.162	.021
	260	.082	.183	.023
	240	.092	.205	.026
	220	.103	.231	.029
	200	.116	.260	.033
	180	.131	.293	.037
14	160	.148	.331	.042
	140	.169	.376	.048
	120	.193	.431	.055
	100	.224	.500	.064
	90	.242	--	--
	80	.263	.587	.075
	70	.289	--	--
	60	.320	.713	.091
	50	.359	--	--
	40	.411	.916	.117
	30	.486	--	--
	20	.607	1.355	.173
	600	.016	.038	.004
	580	.018	.042	.005
	560	.020	.047	.005
	540	.022	.052	.006
	520	.024	.058	.006
	500	.027	.064	.007
	480	.030	.071	.008
	460	.033	.079	.009
15	440	.037	.088	.010
	420	.041	.098	.011
	400	.045	.109	.012
	380	.050	.120	.013
	360	.056	.133	.015
	340	.062	.148	.017
	320	.068	.164	.018
	300	.076	.181	.020
	280	.084	.200	.022

TABLE I (con't)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
	260	.093	.222	.025		280	.094	.238	.024
	240	.103	.246	.027		260	.102	.261	.026
	220	.114	.273	.030		240	.112	.286	.028
	200	.127	.303	.034		220	.123	.314	.031
	180	.141	.338	.038		200	.136	.346	.034
	160	.158	.378	.042		180	.150	.381	.038
	140	.178	.424	.047		160	.166	.423	.042
	120	.202	.482	.054		140	.185	.472	.046
15	100	.231	.553	.062	16	120	.208	.531	.052
	90	.249	--	--		100	.238	.606	.060
	80	.270	.646	.072		90	.255	--	--
	70	.295	--	--		80	.276	.703	.069
	60	.326	.779	.087		70	.301	--	--
	50	.364	--	--		60	.331	.843	.083
	40	.415	.992	.111		50	.369	--	--
	30	.489	--	--		40	.419	1.070	.105
	20	.612	1.462	.163		30	.493	--	--
						20	.615	1.566	.154
	680	.015	.038	.004					
	660	.016	.042	.004		760	.015	.039	.0035
	640	.019	.049	.005		740	.016	.043	.0037
	620	.020	.051	.005		720	.017	.047	.0040
	600	.022	.056	.006		700	.019	.051	.0044
	580	.024	.061	.006		680	.020	.055	.0047
	560	.026	.067	.007		660	.022	.060	.0051
	540	.029	.074	.007		640	.024	.065	.0056
	520	.032	.081	.008		620	.026	.070	.0061
16	500	.035	.088	.009		600	.028	.076	.0065
	480	.038	.097	.010	17	580	.031	.083	.0072
	460	.042	.106	.011		560	.033	.090	.0077
	440	.046	.116	.012		540	.036	.098	.0084
	420	.050	.127	.013		520	.039	.106	.0091
	400	.055	.139	.014		500	.042	.115	.0098
	380	.060	.152	.015		480	.046	.124	.0108
	360	.065	.167	.016		460	.050	.135	.0117
	340	.072	.182	.018		440	.054	.146	.0127
	320	.078	.199	.020		420	.058	.158	.0136
	300	.086	.218	.022		400	.063	.171	.0148

TABLE I (con'd)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
	380	.068	.185	.0160
	360	.074	.200	.0174
	340	.080	.217	.0188
	320	.087	.235	.0204
	300	.094	.254	.022
	280	.102	.276	.024
	260	.110	.299	.026
	240	.120	.325	.028
	220	.131	.354	.031
	200	.143	.387	.034
	180	.157	.424	.037
17	160	.173	.477	.041
	140	.191	.518	.045
	120	.214	.580	.051
	100	.243	.658	.057
	90	.260	--	--
	80	.281	.760	.066
	70	.305	--	--
	60	.335	---	--
	50	.373	--	--
	40	.423	1.145	.100
	30	.496	--	--
	20	.627	1.697	.148
	840	.014	.041	.0031
	820	.015	.044	.0033
	800	.017	.048	.0037
	780	.018	.051	.0040
	760	.019	.055	.0042
	740	.021	.060	.0046
	720	.022	.064	.0048
18	700	.024	.069	.0053
	680	.026	.074	.0057
	660	.028	.080	.0062
	640	.030	.086	.0066
	620	.032	.093	.0071
	600	.035	.100	.0077
	580	.037	.107	.0082
	560	.040	.115	.0088
	540	.043	.123	.0095
	520	.046	.133	.0102
	500	.050	.142	.0111
	480	.053	.153	.0117
	460	.057	.164	.0126
	440	.061	.176	.0135
	420	.066	.189	.0146
	400	.071	.203	.0157
	380	.076	.218	.0168
	360	.081	.233	.0180
	340	.088	.251	.0195
	320	.094	.270	.0208
	300	.101	.290	.0224
	280	.109	.312	.0242
	260	.117	.337	.0260
18	240	.127	.364	.0282
	220	.137	.394	.0304
	200	.149	.427	.0031
	180	.163	.466	.0362
	160	.178	.511	.0395
	140	.197	.564	.0437
	120	.219	.628	.0486
	100	.247	.709	.0548
	90	.265	--	--
	80	.285	.816	.0633
	70	.309	--	--
	60	.338	1.031	.0751
	50	.376	--	--
	40	.425	1.220	.0944
	30	.499	--	--
	20	.619	1.775	1.375
	940	.013	.041	.0027
	920	.014	.043	.0029
	900	.015	.046	.0031
	880	.016	.050	.0033
19	860	.018	.053	.0037
	840	.019	.057	.0040
	820	.020	.061	.0042
	800	.021	.065	.0044
	780	.023	.069	.0048

TABLE I (con'd)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
	760	.024	.074	.0050		1040	.013	.041	.0026
	740	.026	.079	.0054		1020	.014	.043	.0028
	720	.028	.084	.0058		1000	.014	.046	.0028
	700	.030	.090	.0063		980	.015	.049	.0030
	680	.032	.096	.0067		960	.016	.052	.0032
	660	.034	.102	.0071		940	.017	.055	.0034
	640	.036	.109	.0075		920	.018	.059	.0036
	620	.038	.116	.0080		900	.020	.062	.0040
	600	.041	.124	.0086		880	.021	.066	.0042
	580	.044	.132	.0092		860	.022	.070	.0044
	560	.047	.141	.0098		840	.023	.074	.0046
	540	.050	.150	.0105		820	.025	.079	.0050
	520	.053	.160	.0111		800	.026	.084	.0052
	500	.056	.171	.0117		780	.028	.089	.0056
	480	.060	.182	.0126		760	.030	.094	.0060
	460	.064	.194	.0134		740	.031	.100	.0062
	440	.068	.206	.0143		720	.033	.106	.0066
	420	.073	.220	.0153		700	.035	.112	.0070
	400	.077	.234	.0162		680	.037	.119	.0074
	380	.083	.250	.0174		660	.040	.126	.0080
19	360	.088	.267	.0185	20	640	.042	.133	.0084
	340	.094	.285	.0197		620	.044	.141	.0088
	320	.100	.304	.0210		600	.047	.150	.0094
	300	.107	.325	.0225		580	.050	.158	.0100
	280	.115	.348	.0242		560	.052	.166	.0104
	260	.123	.373	.0258		540	.056	.178	.0112
	240	.133	.401	.0280		520	.059	.188	.0118
	220	.143	.432	.0301		500	.063	.199	.0126
	200	.154	.477	.0324		480	.066	.211	.0132
	180	.168	.507	.0353		460	.070	.223	.0140
	160	.183	.554	.0385		440	.074	.237	.0148
	140	.201	.608	.0423		420	.079	.251	.0158
	120	.223	.675	.0469		400	.083	.266	.0166
	100	.251	.760	.0528		380	.089	.282	.0178
	90	.268	--	--		360	.094	.299	.0188
	80	.288	.872	.0606		340	.100	.318	.0200
	70	.312	--	--		320	.106	.338	.0212
	60	.341	1.033	.0717		300	.113	.359	.0226
	50	.378	--	--		280	.120	.383	.024
	40	.428	1.296	.0901		260	.128	.409	.026
	30	.500	--	--		240	.137	.438	.027
	20	.620	1.876	1.305		220	.148	.470	.030

TABLE I (con'd)

T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$	T	(d)	$\frac{U_{max}}{H}$	$\frac{\xi_{max}}{H}$	$\frac{a}{H}$
	200	.159	.506	.032		80	.291	.927	.058
	180	.172	.548	.034		70	.315	--	--
	160	.187	.596	.037		60	.344	1.096	.069
20	140	.205	.653	.041	20	50	.380	--	--
	120	.227	.723	.045		40	.430	1.370	.086
	100	.254	.810	.051		30	.503	--	--
	90	.271	--	--		20	.623	1.984	.125

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